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Self-organized criticality and dislocation damping

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Abstract

Mechanical $1/f$ noise in metallic samples is derived by modelling the relaxation dynamics of dislocation networks as a self-organized critical process. Our predictions compare fairly closely with earlier measurements obtained under diverse experimental conditions.

1. Introduction

Glissile dislocations provide a fairly efficient mechanism of mechanical energy dissipation in crystalline solids subject to periodic strains [1–3]. In real crystals, dislocations break up to form a network of dislocation loops with a characteristic length L_N, which depends on the material and the preparation of each sample. It is usually assumed that the loops are tightly pinned at the network nodes [4]. Defects, such as impurities, vacancies and interstitials, act as a cloud of weaker, mobile pinning points surrounding the network loops with a characteristic zero-temperature binding energy E_C (the Cottrell energy) and an average pin-to-pin distance [3] L_P on the line (with $L_P \ll L_N$). Under stress, the network loops bow out, similarly to an elastic string, being held back at the pinning points. The effective tension of the loop segments connecting two adjacent pins provides the restoring force in the vibrating string model developed by Granato and Lücke [1] (GL model). If the applied stress σ is large enough, *i.e.*

$$\sigma > \sigma_B \simeq E_C/b^2 L_P$$

where b is the modulus of the relevant Burgers vector, the force exerted by the loop segments at the pinning points can overcome the Cottrell binding force and the so-called depinning (or breakaway) process sets in.

Thus, the dislocation loss is made up of two different contributions. (1) The first type is due to the frictional force acting upon each vibrating loop segment as a result of its interaction with the lattice environment [5], and the corresponding dynamic loss is frequency dependent, since the underlying dissipative mechanism is a resonant one. (2) The second type of loss is due to the hysteretic nature of the pinning–depinning mechanism involving loop segments being subject to strong oscillating stresses. For a given stress amplitude, the resulting breakaway loss at zero temperature is expected to be frequency independent.

The distinction between resonant and breakaway loss has to be considered with some caution for at least two reasons. First, for vanishingly small values of the strain angular frequency ω, the decrement function of a crystalline sample ($\Delta(\omega)$) tends apparently to a positive constant [6–8]. Recent measurements [7, 8] have seemed to confirm earlier observations reported by Mason [6], although doubts still remain [9]. In the dynamic loss regime $\sigma < \sigma_B$, $\Delta(0)$ turns out to be independent of σ as well. Such a residual internal friction could not be explained in terms of the GL model. Secondly, in the limit of long forcing cycles, the finite relaxation times of the impurity diffusive dynamics start playing a role [3]; in particular, the pinning–depinning mechanism is thermally activated and the breakaway process is not a clear-cut threshold phenomenon [1]. A strong frequency dependence of the relevant decrement function may ensue at a finite temperature [1, 10].

In a recent paper [11], it was proposed that the relaxation dynamics of a dislocation network interacting with a random impurity pattern can be described as an example of the so-called self-organized criticality (SOC) [12–15]. The basic idea can be sketched as follows. The energy released by depinning of one loop segment propagates along the networks loops and may trigger the depinning of further neighbouring segments, in principle, with no restriction on the number of depinning events, other than the finite size of the sample. Such an "unzipping" mechanism was observed in earlier numerical simulations [16]. A depinned loop segment sweeps through its slip plane until it becomes pinned again by a lattice defect, such as an impurity,

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with sufficiently high binding energy (kinetic energy excess is eventually thermalized). As a result, the entire loop is pinned down into a new configuration. Such a configuration is termed critically stable, because increasing σ by a small amount makes it unstable and a new unzipping mechanism ensues. Furthermore, the time sequence of such a stick-and-slip mechanism is expected to exhibit no intrinsic time-scale. By introducing the notion of a critically stable configuration, we assume implicitly that, under certain circumstances [7, 8], the linear response theory (dynamic loss) and the hysteresis dynamics (breakaway loss) may be inadequate to describe a system as complex as a dislocation network.

In the present article, we elaborate on the possibility of implementing the SOC notion into the vibrating string model for the dislocation dynamics in crystalline solids with low impurity densities.

2. A toy model

Let us introduce dislocation SOC modelling by discussing the simple case of a d-dimensional lattice of pinning points. Without entering into the details of the dislocation network dynamics, we can assume that a local stress $\sigma(\boldsymbol{r}, t)$ is loaded on the pin at $\boldsymbol{r}$. The external stress is increased adiabatically (*i.e.* with a low forcing frequency) by adding a fixed quantity $\delta\sigma$ randomly on the lattice nodes. There is a microscopic limit σ_{th} on the allowed stress on any node. If at a given time the stress on the node $\boldsymbol{r}$ is $\sigma(\boldsymbol{r}, t) > \sigma_{\text{th}}$, then a depinning event occurs: the full amount of stress $\sigma(\boldsymbol{r}, \text{t})$ is transferred in equal parts to d of its $2d$ nearest neighbours in the direction of the applied stress. Thus, the stress $\sigma(\boldsymbol{r}, t)$ is reset to zero, whereas the transferred stress adds to the stress previously stored on the neighbours. If the total stress on these nodes attains the threshold value σ_{th}, further depinning events, *i.e.* stress releases, follow. The sequence of activation events come to an end at a time $t+\tau$, when a new pinning configuration is reached, *i.e.* $\sigma(\boldsymbol{r}, t+\tau) < \sigma_{\text{th}}$ at any $\boldsymbol{r}$. A single stress increase $\delta\sigma$ may trigger off the depinning of a cluster of s connected nodes. This is the unzipping phenomenon observed numerically [16]. This model of dislocation dynamics, valid for materials with high defect densities [11], is certainly oversimplified. Nevertheless, it has the merit of introducing the SOC rationale: in contrast to the GL model, no matter how improved, we have here that stationarity implies criticality.

The comparison of SOC with the theory of critical phenomena has been elucidated in ref. 15. The role of reduced temperature is played by the deviation $\sigma_{\text{th}} - \sigma$, where σ_{th} is a critical stress. The correlation length ξ is the cut-off in the linear cluster size for $\sigma < \sigma_{\text{th}}$. For the sake of generality, the linear size l of a subcritical cluster is related to its size s by the fractal dimension D, such that $s \approx l^D$. The cluster size distribution $D(s)$ is thus characterized by the two power laws.

$$D(s) \approx s^{1-\kappa} \tag{1}$$

$$s_\infty \approx \xi^D \approx \left(1 - \frac{\sigma}{\sigma_{\text{th}}}\right)^{-D\nu} \tag{2}$$

where ν is the critical exponent of the correlation length. Analogously, the lifetimes of the unzipping events (τ) are distributed according to the power law $D(\tau) \approx \tau^{-b}$ and are related to the cluster linear size by the dispersion relationship $\tau \approx l^z$.

Some of the critical exponents introduced above have been determined analytically for isotropic and conservative SOC models such as the present one [14], *i.e.*

$$\kappa = 3 - \frac{2}{d} \qquad z = \frac{1}{3}(d+2) \tag{3}$$

and $b = (\kappa+1)D/z - 2$.

We will now calculate the residual decrement function $\Delta(0)$ under the assumptions of the present SOC model. Since glissile dislocations are likely to move in a slip plane, we confine ourselves to the bidimensional case. Our predictions for $\Delta(0)$ with $d=2$ turn out to depend on the critical exponents κ and z only and, therefore are quite insensitive to the details of the (conservative and isotropic) model adopted here. Following the argument expounded in ref. 11, the lifetime distribution $f(\tau)$ of the unzipping events in a crystalline sample subjected to both a static stress σ with $\sigma < \sigma_{\text{th}}$ and a small periodic stress $\sigma(t)$ with amplitude $\sigma_0 \ll \sigma$ is

$$f(\tau)\, \mathrm{d}\tau = \frac{1}{\ln(\tau_\infty/\tau_0)} \frac{\mathrm{d}\tau}{\tau} \tag{4}$$

The lifetimes τ_0 and τ_∞ set the applicability range of the SOC model. In practice, one tries to estimate the corresponding size range $[s_0, s_\infty]$ through the relationship $\tau \approx s^{z/D}$, which clearly depends on the nature of the sample under study.

To make explicit connections with the vibrating string model, the sample decrement function $\Delta(\omega)$ can be calculated by averaging the single loop segment decrement function $\Delta(\omega, \tau)$ with respect to the lifetime distribution in eqn. (4). According to GL calculations [1], for an overdamped vibrating string of characteristic length L_{P}, we have

$$\Delta(\omega, \tau) = \Delta_0 \Omega \Lambda L_{\text{P}}^2 \frac{\omega\tau}{1+(\omega\tau)^2} \tag{5}$$

where Λ is the total length of dislocation line per unit volume, Ω is an orientation ratio and Δ_0 depends mostly

on the dislocation parameters (*i.e.* the modulus b of the relevant Burgers vector and the effective tension per unit of length C). Therefore, our estimate for $\Delta(\omega)$, *i.e.*

$$\Delta(\omega) = \int_{\tau_0}^{\tau_\infty} \Delta(\omega, \tau)\, f(\tau)\, d\tau$$

$$= \frac{\Delta_0 \Omega \Lambda L_P^2}{\ln(\tau_\infty/\tau_0)} \{\tan^{-1}(\omega\tau_\infty) - \tan^{-1}(\omega\tau_0)\} \qquad (6)$$

gives results quite insensitive to the forcing angular frequency ω over the whole frequency domain $\omega\tau_0 \ll 1$ and $\omega\tau_\infty \gg 1$.

For more details about the derivation of eqns. (4)–(6), the reader is referred to the original work [11]. Here, we limit ourselves to note that eqn. (4) holds well only for the bidimensional case, and that the one free parameter of the present model, *i.e.* the threshold stress σ_{th}, can be determined, at least in principle, experimentally.

3. SOC models of dislocation kink diffusion

The toy model outlined in Section 2 allowed us to interpret satisfactorily some experimental results on low frequency damping in Cu–Be alloys [8]. However, it did not allow us to reproduce the frequency dependence of $\Delta(\omega)$ for samples of low impurity density [2, 6]. The reason is that, for samples with low impurity densities, the SOC mechanism introduced to describe the dislocation–impurity interactions applies only to a small fraction of the dislocation network loops, as shown in the following.

Glissile dislocations are sensitive to the discreteness of the lattice structure. When diffusing in their slip plane, dislocation loops must overcome a periodic structure of Peierls valleys with spacing a (each equivalent to an applied stress σ_P), either by thermal nucleation or (forced) Brownian movement [3, 5]. A dislocation line thrown across N_P+1 Peierls valleys exhibits N_P geometric kinks or antikinks. An applied stress σ pulls the kinks to the left and the antikinks to the right, so that the dislocation line slips in the direction of the stress itself. A kink passes through an antikink but bounces off another kink, and vice versa, the relevant interaction potentials being short ranged. Typically, the repulsive kink–kink potential $U_K(r)$ decays as $Ca^2/2r$ for relative distances r of the order of the kink size or shorter.

If a dislocation segment is pinned between two points separated by N_P Peierls valleys, the N_P geometric kinks (antikinks) repel each other until they reach a uniform distribution with spacing $r_0 = L/N_P$. The orientation of the dislocation line with respect to the Peierls valleys is given by the angle ϕ, where $\sin\phi = a/r_0$. Furthermore, the kink–kink repulsion sets an upper limit to the number of kinks that may be accommodated on a dislocation segment of length L [3]:

$$\frac{N_P}{L} < \frac{l}{a}\left(\frac{ab\sigma_B}{C}\right)^{1/3}$$

Seeger and Schiller [5] worked out the kink model outlined here to calculate the relevant decrement function $\Delta(\omega)$ for a dislocation segment subject to periodic forcing. Their results came very close to the predictions of the vibrating string model. In fact, the observed decrement function at zero frequency $\Delta(0)$ cannot be accounted for by the kink model either.

A further complication arises when one tries to include the lattice substrate interactions. This amounts to introducing a periodic structure of shallow potential valleys (Schottky valleys) intersecting the Peierls valleys with a different spacing a'. Without loss of generality, let us assume here that the two valley families are orthogonal. A Schottky valley is characterized by an effective escape stress (σ_S) which is much smaller than the Peierls stress σ_P. The minimum number N_S of Schottky valleys that a pinned dislocation segment of length L bearing N_P kinks (antikinks) must cross is given by the inequality $N_S \geqslant N_P - \Delta N$. In fact, when N_S is decreased smaller than N_P, a certain number ΔN of kinks are pushed towards the top of the Schottky barriers; as a result, an internal stress is exerted on the pins at the end-point of the dislocation segment. On comparing such an internal stress with the breakaway stress, one concludes that the pinning condition requires

$$\frac{\Delta N}{N_P} \leq \frac{a}{a'}\frac{\sigma_B}{\sigma_S} \qquad (7)$$

Correspondingly, the maximum value of the angle ϕ prior to depinning is

$$\tan\phi = \frac{a}{a'}\frac{1}{1-\Delta N/N_P} \qquad (8)$$

A simple geometric argument [17] leads us to conclude that the fraction of the total dislocation line that can be envisaged as being close to a SOC configuration is

$$\beta \simeq \frac{2}{\pi}\frac{\sigma_B}{\sigma_S} \qquad (9)$$

Here, we have assumed for simplicity that $a/a' = O(1)$ and neglected the kink–kink interactions. When these interactions are included, eqn. (9) becomes modified [17]. The relevant dislocation segments are all oriented according to a critical angle $|\phi| = \tan^{-1}(a/a') \simeq \pi/4$.

The fraction of network loops close to the critical angle can be represented as a one-dimensional chain,

where a gas of short-range interacting quasi-particles, *i.e.* the kinks (or antikinks), are free to move along a periodic, shallow multiwell potential until they hit a pinning point (*e.g.* an impurity) with a higher binding energy. The pinning points are distributed randomly along the chain. Impurity barriers may be overcome by the combined action of an external drive (an applied stress) and the repulsive forces acting, say, between nearest neighbour kinks (antikinks). For a given density of pinning points and kinks, the system exhibits two regimes: (1) when the external force is sufficiently large, the kinks move freely (free flow or "slip" regime, corresponding to $\sigma > \sigma_B$); (2) for low values of the external stress, the spatial distribution of kinks and impurities does not change ("stick" regime). However, even a very small increase in the driving force may produce local avalanches of depinned kinks which travel a distance s before becoming pinned again. Such a critical chain exhibits the typical SOC properties outlined in Section 2, as shown by digital simulation [17].

A simple SOC model for our kink chain can be obtained as a trivial extension of the pinned flux lattice model of ref. 18. For a more detailed modelling of a critical kink chain, the reader is referred to ref. 17. However, for the sake of brevity, let us elaborate on the simulation results of ref. 18. For a wide range of parameter values, the kink displacement distribution $D(s)$ in the "stick" regime is as in eqn. (1) with $\kappa = 2.3 \pm 0.2$. We can then ask ourselves what the probability is that a network loop segment of length L is left behind following an avalanche event. On taking into account that the number of geometric kinks (antikinks) is conserved, one concludes that the corresponding length distribution obeys the power law [17]

$$N_{SOC}(L) = (2\kappa - 3)\frac{2}{L_P}\left(\frac{2L}{L_P}\right)^{-2(\kappa-1)} \tag{10}$$

in the range $[L_P/2, \infty]$. It should be remembered that the length distribution of the remaining fraction $1-\beta$ of thermalized loop segments [1, 3] may be approximated to

$$N_{GL}(L) = \frac{1}{L_P}\exp\left(-\frac{L}{L_P}\right) \tag{11}$$

It follows that, when we take in the GL model the average of the one-segment decrement function $\Delta(\omega, \tau)$ in the dynamic loss regime over the segment length L, the appropriate L distribution function is $\beta N_{SOC}(L) + (1-\beta)N_{GL}(L)$, such that

$$\Delta(\omega) = \beta\Delta_{SOC}(\omega) + (1-\beta)\Delta_{GL}(\omega) \tag{12}$$

For $\kappa = 2.5$, we have

$$\Delta_{SOC}(\omega) = \Delta_0 \Omega \Lambda L_P{}^2 \frac{1}{4}\left\{\frac{\pi}{2} + \tan^{-1}\left(\frac{\omega}{\gamma}\right)\right\} \tag{13}$$

whereas $\Delta_{GL}(\omega)$ is the usual Debye curve plotted in ref. 1. The quantity γ in eqn. (13) denotes the viscous damping constant per unit of dislocation length of the GL model.

4. Conclusions

The comparison between the theoretical predictions in eqns. (12) and (13) and the relevant experimental data [6] proves rather satisfactory [17]. The separation between critical and thermal dislocation loops in eqn. (12) seems to provide a sound interpretation of the residual dislocation friction, independent of the exact value of the critical exponent κ. Thus, more realistic SOC models for a critical dislocation kink chain can be envisaged with as good predictive capabilities as those of the simplified model of ref. 18.

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